Sparse Adaptive Template Matching and Filtering for 2D Seismic Images with Dual-Tree Wavelets and Proximal Methods

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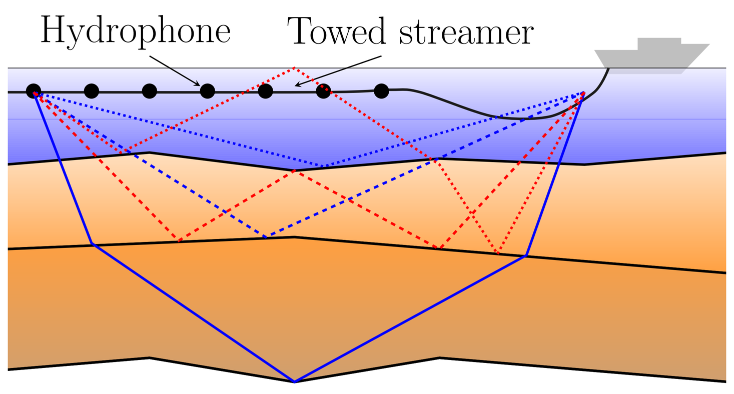
Signal formation model

$$\boxed{z^{(\mathbf{n})} = y^{(\mathbf{n})} + s^{(\mathbf{n})} + b^{(\mathbf{n})} = y^{(\mathbf{n})} + \sum_{j=0}^{J-1} \sum_{p=p'}^{p'+P_j-1} h_j^{(\mathbf{n})}(p) r_j^{(n_t-p,n_x)} + b^{(\mathbf{n})}}$$

- ♣ $\mathbf{n} = (n_t, n_x)$, $n_t \in \{0, \dots, N_t-1\}$ and $n_x \in \{0, \dots, N_x-1\}$
- ♣ P_j tap coefficients, $-P_j+1 \le p' \le 0$, $P = \sum_{j=0}^{J-1} P_j$
- ♣ J approximate templates $r_j^{(\mathbf{n})}$ for the multiple $s^{(\mathbf{n})}$ assumed to be known
- ♣ Inverse problem reduced to time-varying filters $h_j^{(\mathbf{n})}(p)$ estimation
- ♣ Additional sparsity constraints taken into account.

Principles of seismic acquisition



Reflections on different layers (primaries in blue), and reverberated disturbances (multiples in dotted and dashed red).

Proximity operator

$\mathcal{H}$: Hilbert space, $\psi : \mathcal{H} \to \mathcal{H}$: lower semi-continuous convex function

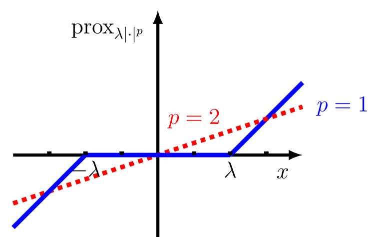
$$\operatorname{prox}_\psi : \mathcal{H} \to \mathcal{H} : u \mapsto \operatorname*{argmin}_{v \in \mathcal{H}} \frac{1}{2}\|v-u\|^2 + \psi(v)$$

C : convex set in $\mathcal{H}$ $\hookrightarrow$ $\operatorname{prox}_{\iota_C}(x) = \underbrace{\Pi_C(x)}_{\text{projection onto } C} \qquad \forall x \in \mathcal{H}$

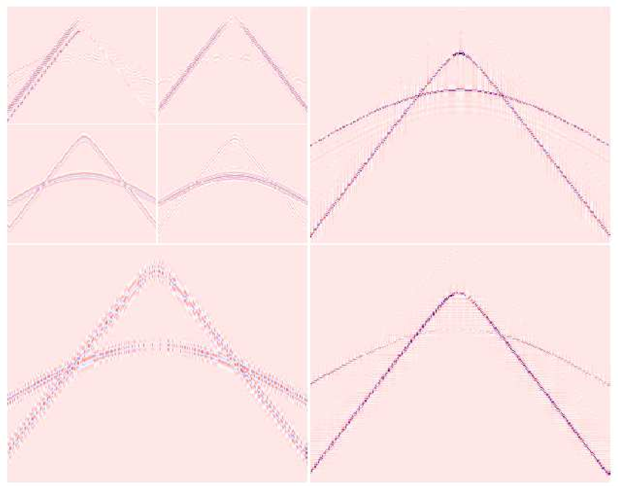
$(\forall x \in \mathbb{R})$

a) $\operatorname{prox}_{\lambda|\cdot|^2}(x) = \underbrace{\frac{1}{1+2\lambda}}_{\text{"Wiener" filter}} x$

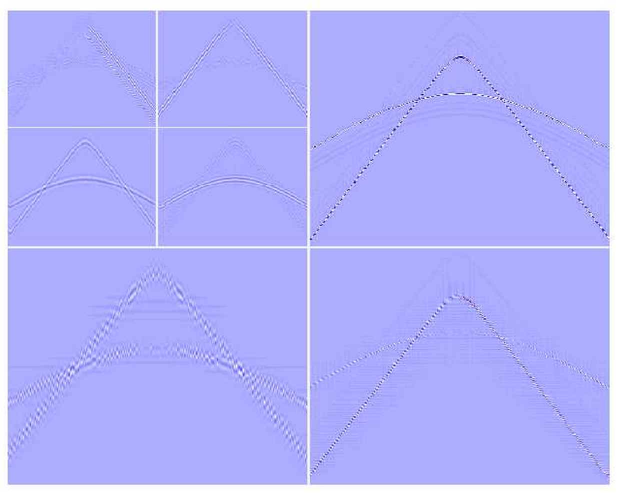
b) $\operatorname{prox}_{\lambda|\cdot|}(x) = \underbrace{\operatorname{sign}(x)\max(|x|-\lambda, 0)}_{\text{shrinkage operator}}$



Dual-tree M-band wavelet



Primal coefficients



Dual coefficients

MAP estimation

$$\boxed{\operatorname*{minimize}_{y \in \mathbb{R}^{N_t N_x}, \mathbf{h} \in \mathbb{R}^{N_t N_x P}} \|z - \mathbf{R}\mathbf{h} - y\|^2 + \iota_D(Fy) + \iota_C(\mathbf{h})}$$

- ♣ $\mathbf{R}$: defined from J templates
- ♣ $F \in \mathbb{R}^{K \times N_t N_x}$: hybrid dual-tree wavelet $\hookrightarrow$ $x = Fy \in \mathbb{R}^K$
- ♣ $D = D_1 \times \ldots \times D_{2\mathcal{L}}$, $\quad \{1, \dots, K\} = \bigcup_{l=1}^{2\mathcal{L}} \mathbb{K}_l$, $\quad \beta_l \in]0, +\infty[$

$$\forall l \in \{1, \ldots, \mathcal{L}\}, \quad D_l = \left\{ (x_k)_{k \in \mathbb{K}_l \cup \mathbb{K}_{l+\mathcal{L}}} : \underbrace{\sum_{k \in \mathbb{K}_l} |x_k| \le \beta_l}_{\text{primal coefficients}} \text{ and } \underbrace{\sum_{k \in \mathbb{K}_{l+\mathcal{L}}} |x_k| \le \beta_l}_{\text{dual coefficients}} \right\}$$

- ♣ $C = C_1 \cap C_2 \cap C_3$
- ♣ Slow variations of the filters:
  1. Along time $C_1 = \left\{ \mathbf{h} \mid \left| h_j^{(n_t+1,n_x)}(p) - h_j^{(n_t,n_x)}(p) \right| \le \varepsilon_{j,p}^{n_x} \right\}$
  2. Along sensors $C_2 = \left\{ \mathbf{h} \mid \left| h_j^{(n_t,n_x+1)}(p) - h_j^{(n_t,n_x)}(p) \right| \le \varepsilon_{j,p}^{n_t} \right\}$
- ♣ Concentration metrics on the filters ($\lambda \in]0, +\infty[$)

$$C_3 = \left\{ \mathbf{h} \in \mathbb{R}^{N_t N_x P} \mid \ell_{1,2}(\mathbf{R}\mathbf{h}) \le \lambda \right\}$$

- ♣ $\forall d \in \mathbb{R}^{N_t \times N_x}$, $\ell_{1,2}(d) = \sum_{n_x=0}^{N_x-1} \left(\sum_{n_t=0}^{N_t-1} (d^{(n_t,n_x)})^2 \right)^{1/2}$

M+L FBF algorithm

Set $\gamma^{[i]} \in [\epsilon, \frac{1-\epsilon}{\vartheta}]$.
Initialization:
$y^{[0]} \in \mathbb{R}^{N_t N_x}$, $\mathbf{h}^{[0]} \in \mathbb{R}^{N_t N_x P}$, $v^{[0]} \in \mathbb{R}^K$, $u^{[0]} \in \mathbb{R}^{N_t N_x P}$
Iterations:
for $i = 0, 1, \ldots$ **do**

Gradient computation

$s_1^{[i]} = y^{[i]} - \gamma^{[i]} \left(\mathbf{R}\mathbf{h}^{[i]} + y^{[i]} - z + F^* v^{[i]} \right)$
$t_1^{[i]} = \mathbf{h}^{[i]} - \gamma^{[i]} \left(\mathbf{R}^\top (\mathbf{R}\mathbf{h}^{[i]} + y^{[i]} - z) + u^{[i]} \right)$

Projection computation

$s_2^{[i]} = v^{[i]} + \gamma^{[i]} F y^{[i]}$ and $w_1^{[i]} = s_2^{[i]} - \gamma^{[i]} \Pi_D ((\gamma^{[i]})^{-1} s_2^{[i]})$
$t_2^{[i]} = u^{[i]} + \gamma^{[i]} \mathbf{h}^{[i]}$ and $w_2^{[i]} = t_2^{[i]} - \gamma^{[i]} \Pi_C (\gamma^{[i]})^{-1} t_2^{[i]})$

Averaging

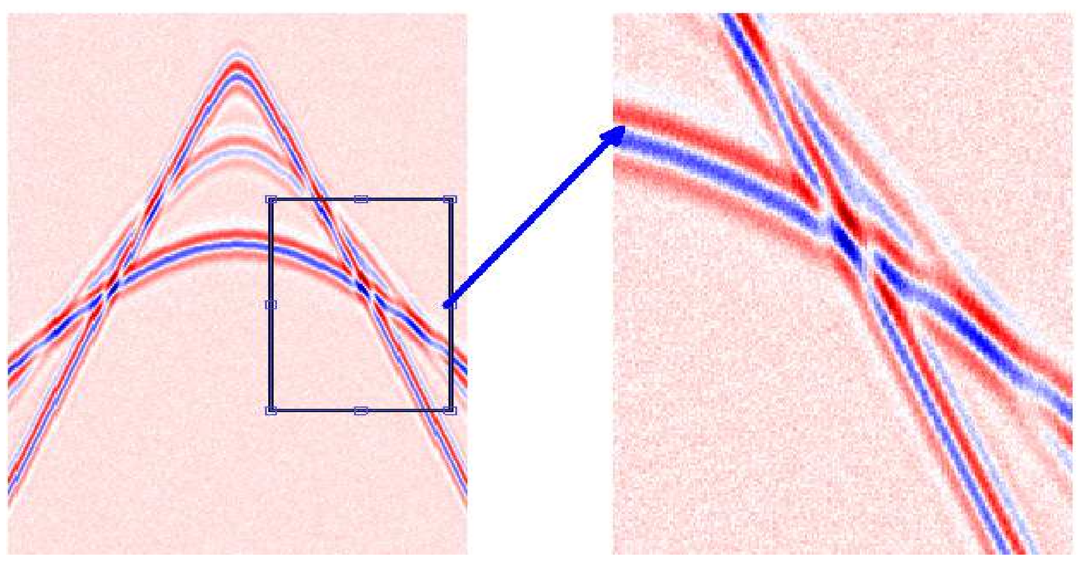
$q_1^{[i]} = w_1^{[i]} + \gamma^{[i]} F s_1^{[i]}$ and $v^{[i+1]} = v^{[i]} - s_2^{[i]} + q_1^{[i]}$
$q_2^{[i]} = w_2^{[i]} + \gamma^{[i]} t_1^{[i]}$ and $u^{[i+1]} = u^{[i]} - t_2^{[i]} + q_2^{[i]}$

Update

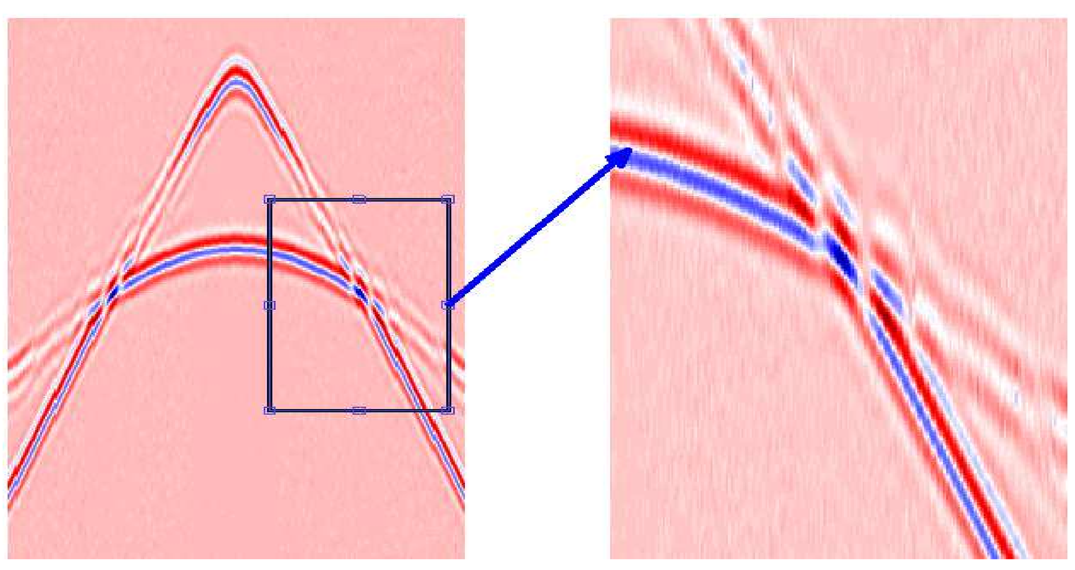
$y^{[i+1]} = y^{[i]} - \gamma^{[i]} \left(\mathbf{R} t_1^{[i]} + s_1^{[i]} - z + F^* w_1^{[i]} \right)$
$\mathbf{h}^{[i+1]} = \mathbf{h}^{[i]} - \gamma^{[i]} \left(\mathbf{R}^\top (\mathbf{R} t_1^{[i]} + s_1^{[i]} - z) + w_2^{[i]} \right)$

end for

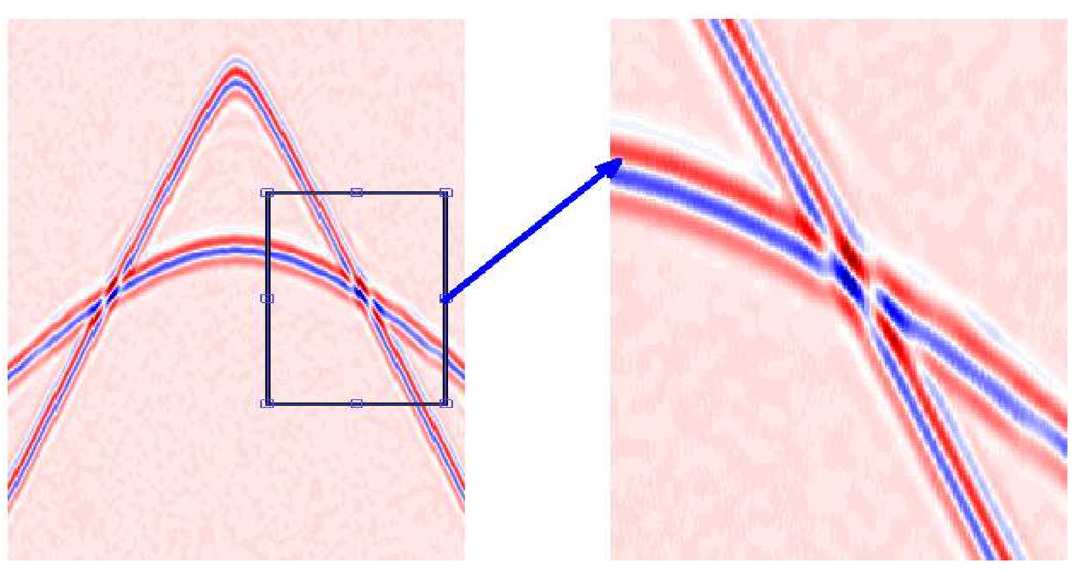
Synthetic data



Observed data z ($\sigma = 0.04$)

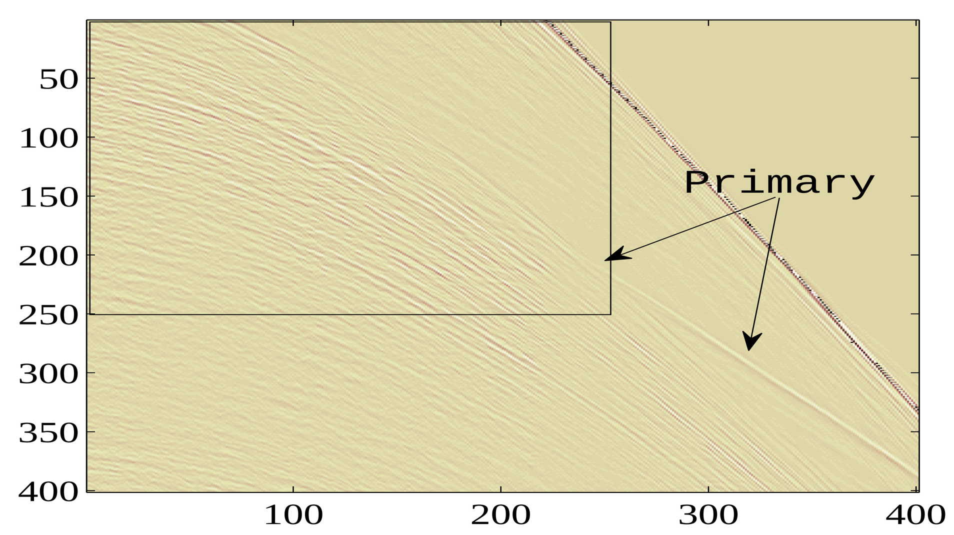


Estimated $\widehat{y}$ by 1D method (SNR = 7.08 dB)

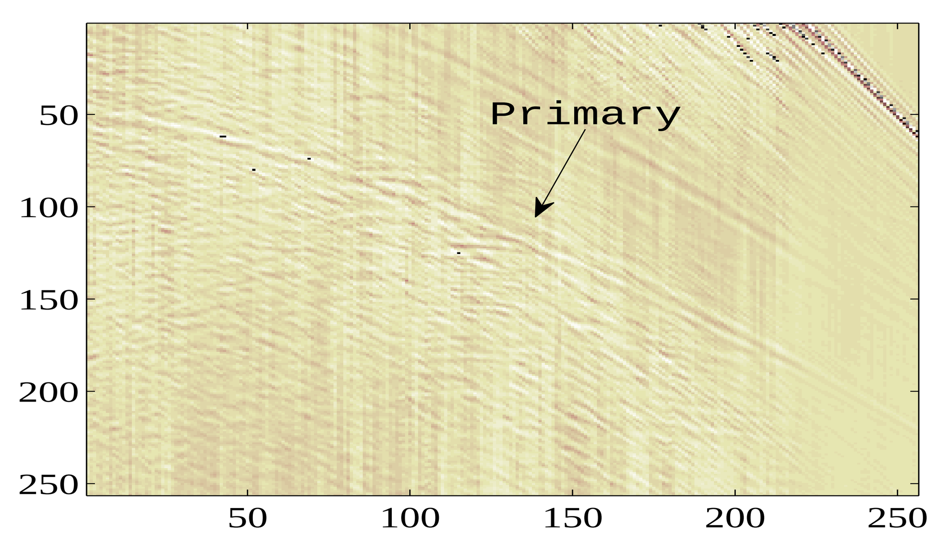


Estimated $\widehat{y}$ by 2D method (SNR = 17.17 dB)

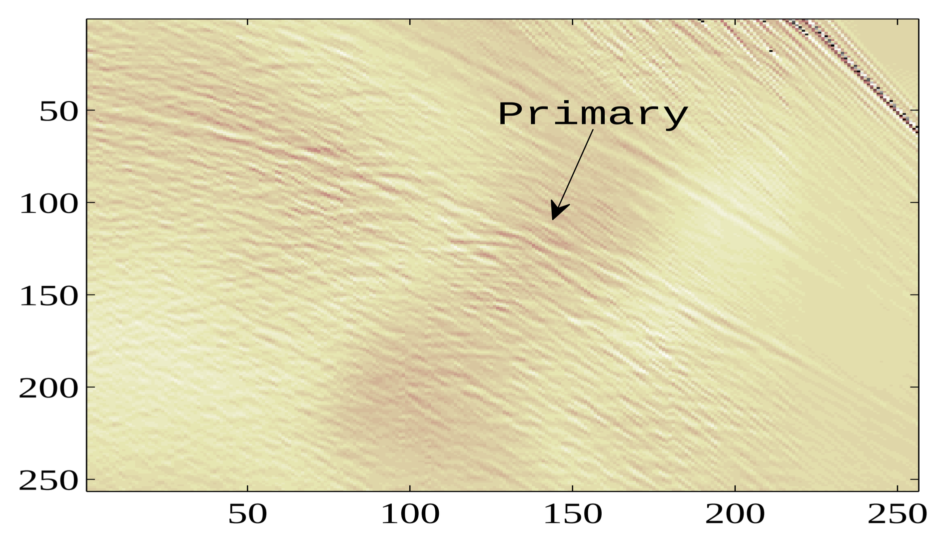
Real data



Observed data z



Estimated $\widehat{y}$ by 1D method



Estimated $\widehat{y}$ by 2D method

Numerical experiments

- ✓ Filter lengths: $P_0 = 4$, $P_1 = 4$ ($J = 2$)
- ✓ Iterations: 10000 (stopping if $\|y^{[i+1]} - y^{[i]}\| < 10^{-6}$)
- ✓ Constraint choice: $\varepsilon_{j,p}^{n_x} = 0.05$ and $\varepsilon_{j,p}^{n_t} = 0.0001$ $\forall (j,p)$.

	$F \setminus \sigma$	0.04	0.08	0.16
y	orthogonal basis	13.93	11.05	7.45
	shift-invariant frame	15.51	13.21	10.71
	M-band dual-tree	17.17	15.60	12.67
s	orthogonal basis	7.79	7.09	4.65
	shift-invariant frame	7.74	6.64	5.26
	M-band dual-tree	9.82	9.37	7.01

SNR for the estimations of y and s in dB considering different wavelet transforms F and three noise levels.

Key message

- ✓ Adaptive filtering with convex optimization,
- ✓ Large choice in sparse 2D wavelet transforms,
- ✓ Efficiency of the low-redundant M-band dual-tree.

References

[1] M. Q. Pham, L. Duval, C. Chaux, and J.-C. Pesquet, "A primal-dual proximal algorithm for sparse template-based adaptive filtering: Application to seismic multiple removal", *IEEE Trans. Signal Process.*, vol. 62, no. 16, pp. 4256–4269, Aug. 2014.

[2] C. Chaux, L. Duval, and J.-C. Pesquet, "Image analysis using a dual-tree M-band wavelet transform", *IEEE Trans. Image Process.*, vol. 15, no. 8, pp. 2397–2412, Aug. 2006.

[3] P. L. Combettes and J.-C. Pesquet, "Primal-dual splitting algorithm for solving inclusions with mixtures of composite, Lipschitzian, and parallel-sum type monotone operators", *Set-Valued Var. Anal.*, vol. 20, no. 2, pp. 307–330, Jun. 2012.