Coherent noise removal in seismic data with dual-tree $M$-band wavelets

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**ABSTRACT**

Seismic data and their complexity still challenge signal processing algorithms in several applications. The advent of wavelet transforms has allowed improvements in tackling denoising problems. We propose here coherent noise filtering in seismic data with the dual-tree $M$-band wavelet transform. They offer the possibility to decompose data locally with improved multiscale directions and frequency bands. Denoising is performed in a deterministic fashion in the directional subbands, depending of the coherent noise properties. Preliminary results show that they consistently better preserve seismic signal of interest embedded in highly energetic directional noises than discrete critically sampled and redundant separable wavelet transforms.

**Keywords:** Dual-tree $M$-band wavelets, Seismic processing, Directional noise filtering.

1. **INTRODUCTION**

The complexity of seismic data and their interpretation have contributed to the development of several efficient signal processing tools. One of the most interesting methods arising from geophysical problems is the wavelet transform, first attributed to Jean Morlet et al. in the eighties. Both the traditional continuous and discrete wavelet transforms (CWT and DWT respectively) have been successfully applied in several areas of geosciences, and more generally in theoretical and applied signal and image processing problems. Their performance is often demonstrated in restoration contexts (e.g. denoising or deconvolution), where signals are corrupted by relatively basic degradation models, including convolutive filtering or additive/multiplicative random noises. Objective results may then be assessed by Monte-Carlo simulations carried out on template signals, often based on estimated degradation parameters (e.g. noise variance). For other contexts where the processing itself degrades signals (for instance compression), additional subjective evaluations are necessary, since the objective assessment of degradations models becomes less correlated to subjective interpretation. Geosciences offer a handful of examples where perturbations are, in general, hardly modeled by relatively generic information on wave propagation. In these occurrences, disturbances that hinder interpretation are very often data dependent, and require customized case-based processing.

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Such a situation arises for coherent noise removal in seismic data. Coherent noise here denotes structured signals present in seismic records, which are unwanted since they hamper subsequent geophysical data processing and interpretation. In new petroleum provinces such as foothill areas, these disturbances are highly energetic and complex to model. Consequently, traditional filtering methods are often not able to eliminate them and to reveal important geophysical information. We propose in this work a practical use of dual-tree $M$-band wavelets for coherent noise removal. Their ability to locally decompose images in accurate frequency bands and selected directions provides encouraging subjective results for these problems. In Section 2 we state the problem of coherent noise in the context of seismic images and recall related works. Section 3 briefly reviews the dual-tree $M$-band wavelet decomposition. Section 4 finally focuses on its application to coherent noise filtering in seismic data, followed by concluding remarks in Section 5.

2. SEISMIC PROBLEM STATEMENT AND BACKGROUND WORK

Seismic exploration aims at providing information about the ground substructures. This information is indirectly addressed by disturbances, artificially created by seismic energy sources. The disturbances propagate through the ground, where geophysical strata reflect the spreading wave front. Portions of the reflected waves are then collected by sensors, often situated near the ground surface. The one-dimensional signal acquired by a single sensor is called a seismic trace. In the simplest convolutive earth model, a trace is a time-based signal composed of the generated disturbance convolved with the reflection coefficients at the strata interfaces. Seismic processing is the task of inferring substructure locations and properties from the collected signals, with the help of geological models. Seismic signals generally decrease in energy as the wave fronts propagate deeper and are scattered by subsurface heterogeneities. The signals are also corrupted by several noise sources that reduce the possibility to detect essential information such as strata or faults. Seismic data filtering is thus a prominent task in seismic processing, especially as exploration aims at imaging deeper targets, in geologically disturbed zones. Filtering may be carried out on the 1-D seismic signal. Meanwhile, seismic data can be re-arranged to account for the correlation between signals recorded by neighboring sensors. For instance, seismic shot gathers are represented as images in which each column is formed by seismic traces, ordered after the regular sensor array location. In a simple earth model based on horizontal layers, reflections appear like hyperbolas whose characteristics reveal information on the wave celerity in geological strata and their depth.\(^2\) Ground-roll is an example of coherent noise that strongly disturbs meaningful seismic information. It depends on the acquisition parameters of the seismic records and on the topography of the near surface. In the former model, these surface waves result from non-reflective guided waves in the poorly consolidated first layer (the weathered zone). They usually possess a lower frequency content than their reflection counterparts and display an apparent linear, highly energetic behavior on seismic images. They are traditionally removed using Fourier-based techniques and simple modeling.\(^2\)

The exploration of new petroleum provinces is of strategical importance for the oil industry. For instance, a large percentage of oil discoveries from 1920 were located in folded and thrust belt areas, where exploration remains challenging, from seismic acquisition to depth imaging. Surface waves, which represent most of the coherent noise energy in foothills areas, are very complex to eliminate by classical Fourier methods, since due to rugged topography, surface waves do not appear any longer as linear events. Besides, these topography changes as well as heterogeneities of the near surface scatter the propagating surface waves generating hyperbolas if the scattering locations are not along the recorded line. Since such areas and the resulting disturbances are difficult to model, noise elimination requires robust methods able to address its locally specific frequency content and directional properties.
Figure 1. Original noisy seismic data in shot gather.
Figure 2. Examples of subband coefficients at different scales and frequency contents: (a-b), (c-d) negatively positively and oriented coefficients; (e-f) coefficients' energy computed from the two dual-trees at given scale and frequency band.
Figure 3. Directional noise removed from local mute of dual-tree coefficients.
Figure 4. Filtered residual.
Multiscale analysis with classical DWT has shown very effective both theoretically and practically, but it has been recognized that classical wavelets, even if improving on Fourier techniques\(^3\), are not always robust enough, especially with strong or aliased coherent noise. A majority of proposed solutions relies on adding redundancy to the transform, using redundant wavelets.\(^4\)\(^5\) New tools generalizing wavelets are recently been used in this context, for instance curvelets, whose ability to isolate different directions at different scales have shown useful. All these methods incur a relatively high redundancy cost, which hamper the tractability of huge data set processing. Another candidate for low-redundancy, local directional filtering is the dual-tree discrete wavelet transform\(^6\) two classical wavelet trees are developed in parallel, with filters forming (approximate) Hilbert pairs. The design of dual-tree filters is addressed through a Hilbert pair formulation for the “dual” wavelets.\(^7\) This transform has already been applied to geophysical data processing, for instance to random noise filtering.\(^8\)\(^9\)\(^10\)\(^11\)\(^12\)\(^13\)\(^14\) They illustrate that different directions can be extracted from the present transform, since high-pass wavelets select opposite directions for each tree. Since the present paper focuses on applications, we refer to \(^6\)\(^12\)\(^14\) for further results on dual-tree wavelet transforms.

3. M-BAND WAVELETS AND HILBERT PAIRS

Let \( M \) be an integer greater than or equal to 2. An \( M \)-band multiresolution analysis of \( L^2(\mathbb{R}) \) is defined by one scaling function \( \psi_0 \in L^2(\mathbb{R}) \) and \( (M - 1) \) mother wavelets \( \psi_m \in L^2(\mathbb{R}), \ m \in \{1, \ldots, M - 1\} \),\(^13\) solutions of the following scaling equations:

\[
\forall m \in \{0, \ldots, M - 1\}, \quad \frac{1}{\sqrt{M}} \psi_m(\frac{t}{M}) = \sum_{k=-\infty}^{\infty} h_m[k] \psi_0(t - k), \quad (1)
\]

where the sequences \( (h_m[k])_{k \in \mathbb{Z}} \) are square integrable. A “dual” \( M \)-band multiresolution analysis is defined by a scaling function \( \psi_0^H \) and mother wavelets \( \psi_m^H, \ m \in \{1, \ldots, M - 1\} \) related to the functions \( \psi_m \) by an Hilbert pair relationship. More precisely, the dual mother wavelets will be obtained by an Hilbert transform from the “primal” wavelets \( \psi_m, \ m \in \{1, \ldots, M - 1\} \). In the Fourier domain, the Hilbert transform reads:

\[
\forall m \in \{1, \ldots, M - 1\}, \quad \widehat{\psi_m^H}(\omega) = - i \text{sign}(\omega) \widehat{\psi_m}(\omega). \quad (2)
\]

where the signum function \( \text{sign} \) is defined as:

\[
\text{sign}(\omega) = \begin{cases} 
1 & \text{if } \omega > 0 \\
0 & \text{if } \omega = 0 \\
-1 & \text{if } \omega < 0.
\end{cases} \quad (3)
\]

These dual wavelets also satisfy two-scale equations similar to Eq. (1) with the square integrable sequences \( (g_m[k])_{k \in \mathbb{Z}} \). The general representation in terms of filter banks is shown in Fig. 5. Figure 6 represents the 1-D basis functions obtained\(^12\) with \( M = 8 \) bands. The corresponding 2-D spatial wavelets are represented in Figure 7. They illustrate that different directions can be extracted from the present transform, since high-pass wavelets select opposite directions for each tree. Since the present paper focuses on applications, we refer to \(^6\)\(^12\)\(^14\) for further results on dual-tree wavelet transforms.
4. APPLICATIONS TO COHERENT SEISMIC NOISE FILTERING

The aim of this work is to reveal reflection signals by locally removing band-pass and directional noises. We have performed tests on a 750 × 300 seismic image partially displayed in Figure 1. Each column of this image (displayed rotated by 90 degrees) represents a 1-D signal acquired by a sensor. As a consequence, the horizontal direction of the image represents a spatial extent for sensors aligned on the ground surface. A seismic source pulse with strong energy is generated around the center of the acquisition line. The resulting waves are reflected and refracted by the underground substructures. They arrive at varying times depending on the sensor distance to the seismic source. The vertical direction thus represents a time extent related to the distance and the propagation velocity in the considered medium. In these data, the first transmitted waves reach the most-left sensor (with index 1) after 180 time units approximately. The first chevron-like lines with an apparent slope of ±30 degrees correspond to refracted waves. The acute cone originating from the same apex represents a highly energetic, directional and coherent signal (ground-roll) from the near surface, usually considered as noise since in general it does not bear significant geological information. Its high amplitudes often obfuscate important information reflecting on subsurface interfaces, which have hyperbola shapes in simple media. Traditional methods rely on the apparent velocity of ground-roll, based on 2-D Fourier (F-K filtering) or Radon transforms.\textsuperscript{2,15} They consist in the projection of the data onto a more appropriate basis and the elimination (local mute) of transformed coefficients related to the unwanted noise. These methods often reach their limits on complex media, due to the local origins of the coherent noise. Standard\textsuperscript{16} or redundant\textsuperscript{4,5} wavelet transforms have been proposed, with a limit due to their lack of directionality and computational cost for the latter. A recent work has proposed a curvelet domain thresholding technique.\textsuperscript{17} The infinite support of curvelets and their redundancy may limit their performance in some applications. Our approach is based on a dual-tree $M$-band wavelet transform with elimination of patches of wavelets coefficients in each direction and scale of the transformed domain. Figure 2 displays, column-wise, examples of subband coefficients at different scales and frequency content of the considered seismic image: Fig. 2-(a,b) and 2-(c,d) represent the coefficient obtained from the negatively and positively oriented 2-D wavelets respectively. Fig. 2-(e,f) represents the cumulated energy from the negative and positive coefficients. In the selected subbands and scales, strong energetic features related to the ground-roll are visible, and allow local coefficient cancellation in the transformed domain.

After coefficient selection in the transformed domain, based on the noise’s orientation, the directional noise...
Figure 6. Monodimensional 8-band dual-tree wavelets: (Top) primal scaling function $\psi_0$ and seven wavelets $\psi_k$; (Bottom) dual scaling function $\psi^H_0$ and seven wavelets $\psi^H_k$. 
Figure 7. Bidimensional 8-band dual-tree wavelets: (Left) negatively oriented scaling function $\psi_0$ and 63 wavelets $\psi_k$; (Right) positively oriented scaling function $\psi_0^\text{H}$ and 63 wavelets $\psi_k^\text{H}$.

removed is displayed in Figure 3. Apparently, non visible scattered noise on Fig. 1 with positive directions between the 200 and 250 spatial indices has also been removed. The performance of the method is evident from Fig. 4. Although it is difficult to infer the true nature of the seismic data, interesting reflection signals have emerged beneath the noise cone, for instance around 100-130 horizontal and 100-250 vertical indices. Interestingly, newly revealed reflections extend seismic information already apparent outside the noise cone. The results have been subjectively validated by geophysicists.

5. CONCLUSIONS

We devised coherent, directional seismic noise removal with dual-tree $M$-band wavelets, combining the features of multi-channel filter banks and improved directionality due to their Hilbert pair structure. This method appears efficient for ground-roll filtering, especially in complex areas where the coherent noise is highly energetic and possesses local directional changes. This method is attractive since it only generates a two-fold redundancy. It is being further validated on other types of seismic data exhibiting similar directional noises.

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