0000 SEISMIC DATA ANALYSIS WITH DUAL-TREE M-BAND WAVELET TRANSFORMS

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Summary

The complexity of seismic data still challenges signal processing algorithms in several applications. The rediscovery of wavelet transforms by J. Morlet *et al.* has allowed improvements in addressing several data representation (local analysis, compression) and restoration problems. However, despites their achievements, traditional approaches based on discrete and separable (both for computational purposes) wavelets fail at efficiently representing directional or higher dimensional data features, such as line or plane singularities, especially in severe noise conditions. Subsequent extensions to wavelets (multiscale pyramids, curvelets [YaTrHe04][Do06], contourlets, bandlets) have recently generated tremendous theoretical and practical interests. They feature local and multiscale properties associated with a certain amount of redundancy, which may represent an issue for huge datasets processing.

We propose here seismic data processing based on dual-tree M-band wavelet transforms [ChDuPe06]. They combine:

- orthogonal M-band filter banks which better separate frequency bands in seismic data than wavelets, due to the increased degrees of freedom in the filter design,
- Hilbert transform and complex representation of seismic signals which have been effective, especially for attributes definition,

with a relatively low redundancy (a factor of two). These transforms have been successfully applied to random noise removal in traditional and remote sensing imagery.

We apply them to seismic data and address their potential for local slope analysis and coherent noise (ground-roll) filtering.

Introduction

The complexity of seismic data has contributed to the development of efficient processing tools. One of the most interesting methods arising from geophysical problems is the wavelet transform (WT), contributed by Jean Morlet *et al.* in the eighties. Wavelets have proved successful in several areas of geosciences. Meanwhile, new tools generalizing wavelets have been used in geosciences, for instance in [MiKiHo05], where complex wavelets [SeBaKi05] are applied to migration.

On the one hand, data compression based on M-channel filter banks (FB) have been considered in seismics [DuRø00]. On the other hand, Hilbert transform and complex representation of seismic signals are quite effective, especially for attributes definition. We propose here the use of dual-tree M-band wavelets, which combines multi-channel FBs and Hilbert transforms, for coherent noise removal. We develop some of their properties, including improved directional analysis. A practical application is performed on the denoising of a seismic image. We show that we are able to recover useful seismic information even in very noisy environment.

1 Background work

Multiscale analysis with classical discrete wavelet transform (DWT) has shown very effective both theoretically and practically, but it has been recognized that classical wavelets are not always robust enough, especially with strong or aliased coherent noise [GaDuDu02]. A majority of proposed solutions relies on adding redundancy to the transform. One of the most promising decomposition is the *dual-tree* discrete wavelet transform [SeBaKi05]: two classical wavelet trees are developed in parallel, with filters forming (approximate) Hilbert pairs. The design of dual-tree filters is addressed through a Hilbert pair formulation for the "dual" wavelets. Generalizations to 2D dual-tree M-band wavelet decompositions, proposed in [ChDuPe06], are briefly reviewed in the next section.

2 M-band wavelets and Hilbert pairs

Let M be an integer greater than or equal to 2. An M-band multiresolution analysis of $\mathrm{L}^2(\mathbb{R})$ is defined by one scaling function $\psi_0 \in \mathrm{L}^2(\mathbb{R})$ and $(M-1)$ mother wavelets $\psi_m \in \mathrm{L}^2(\mathbb{R})$, $m \in \{1,\dots,M-1\}$. For standard wavelets, $M=2$, and the generalization in terms of filter banks is shown in Fig. 1. A "dual" M-band multiresolution analysis is defined by a scaling function ψ_0^{H} and mother wavelets ψ_m^{H}, $m \in \{1,\dots,M-1\}$. More precisely, the mother wavelets will be obtained by a Hilbert transform from the "original" wavelets ψ_m, $m \in \{1,\dots,M-1\}$. In the Fourier domain, the Hilbert transform reads:

$$\forall m \in \{1,\dots,M-1\}, \qquad \widehat{\psi}_m^{\mathrm{H}}(\omega) = -\imath\,\mathrm{sign}(\omega)\widehat{\psi}_m(\omega). \tag{1}$$

Figure 2 shows the 1D basis functions obtained [ChDuPe06] with $M=8$ bands. The primal and dual wavelets are in quadrature, as can be seen from their pairwise symmetry/antisymmetry. The corresponding 2D spatial wavelets are represented in Figure 3. They illustrate that different directions can be extracted from the present transform, since band-pass and high-pass wavelets select opposite directions for each tree.

3 Preliminary application to local directional analysis

The 2D M-band dual-tree wavelets represented in Figure 3 are used for local frequency and slope estimation. The data represented in Figure 4-(Top) contains varying orientation with different frequency content. It undergoes an analysis with two wavelets from the dual-tree having corresponding frequency content and scale with opposite directions. Figures 4-(Middle/Bottom) represent the resulting energy map. High amplitude coefficients are obtained where the slope and frequency contents of the data match those of the wavelets. Similar maps are obtained for each 2D wavelet.

4 Application to ground-roll filtering

In this application, reflection signals are revealed by locally removing directional noises. Traditional methods rely on the apparent velocity of ground-roll, based on 2D Fourier (F-K filtering) or Radon transforms. These methods often reach their limits in complex media, due to the local origins of the coherent noise.

Standard or undecimated [GaDuDu02] discrete wavelet transforms have been proposed, with a limit due to their lack of directionality and computational cost for the latter. A recent work has proposed a curvelet domain thresholding technique [YaTrHe04]. The infinite support of curvelets and their redundancy may limit their performance in some applications. Our approach is based on a dual-tree M-band wavelet transform with cancellation of patches of wavelets coefficients in each direction and scale of the transformed domain.

We have performed preliminary tests on a 750×300 seismic image partially displayed on top of Figure 5. In these data, the first transmitted waves reach the most-left sensor (with index 1) after 180 time units approximately. The first chevron-like lines with an apparent slope of ± 45 degrees correspond to refracted waves. The acute cone originating from the same apex represents a highly energetic, directional and coherent signal (ground-roll) from the near surface, usually considered as noise since in general it does not bear significant geological information. Their high amplitudes often obfuscate important information reflecting on subsurface interfaces, which have hyperbola shapes in simple media. After coefficient selection on the transformed domain, based on the noise's apparent direction, the directional noise removed is displayed in the middle of Figure 5. The performance of the method is apparent from the bottom of Fig. 5. Although it is difficult to infer the true nature of the seismic data, interesting reflection signals have emerged beneath the noise cone, for instance around 100-130 horizontal and 100-250 vertical indices.

Conclusions

We devised coherent, directional seismic noise removal with dual-tree M-band wavelets, combining the features of multi-channel filter banks and improved directionality due to its Hilbert pair structure. This method appears efficient for filtering, especially in complex area where the coherent noise is highly energetic and possesses local directional changes. This method is attractive since it only generates a two-fold redundancy. Future work focuses on its use for seismic data compression.

References

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Illustrations

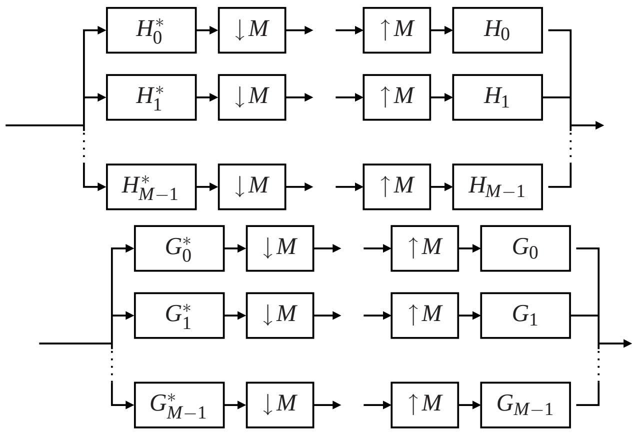


Figure 1: An Hilbert pair of analysis/synthesis M-band para-unitary filter banks.

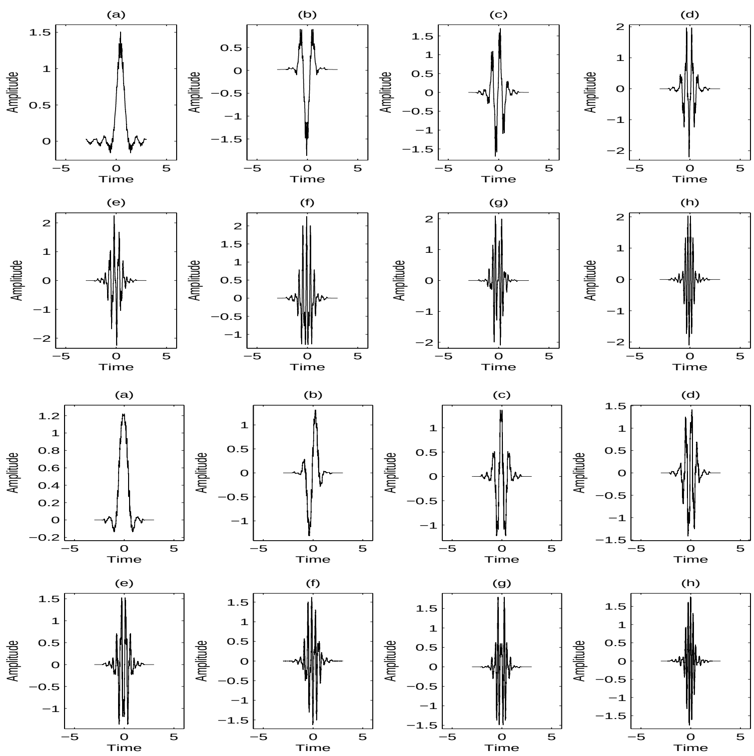


Figure 2: Monodimensional 8-band dual-tree wavelets: (Top) primal scaling function ψ_0 and seven wavelets ψ_k; (Bottom) dual scaling function ψ_0^{H} and seven wavelets ψ_k^{H}.

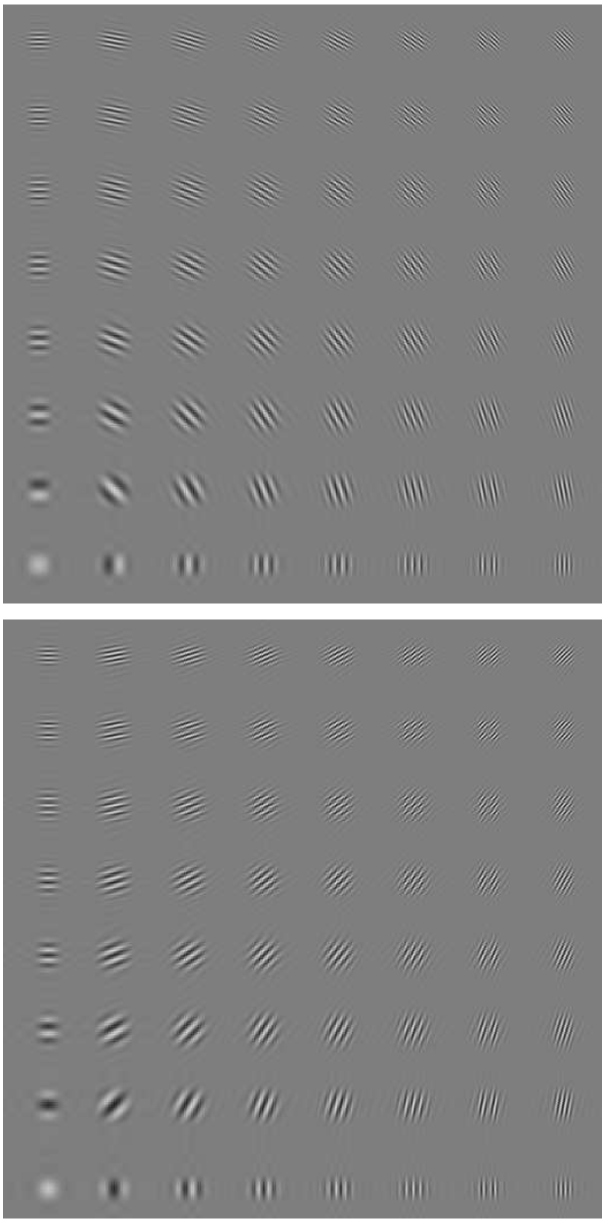


Figure 3: Bidimensional 8-band dual-tree wavelets: (Top) primal scaling function ψ_0 and 63 wavelets ψ_k; (Bottom) dual scaling function ψ_0^{H} and 63 wavelets ψ_k^{H}.

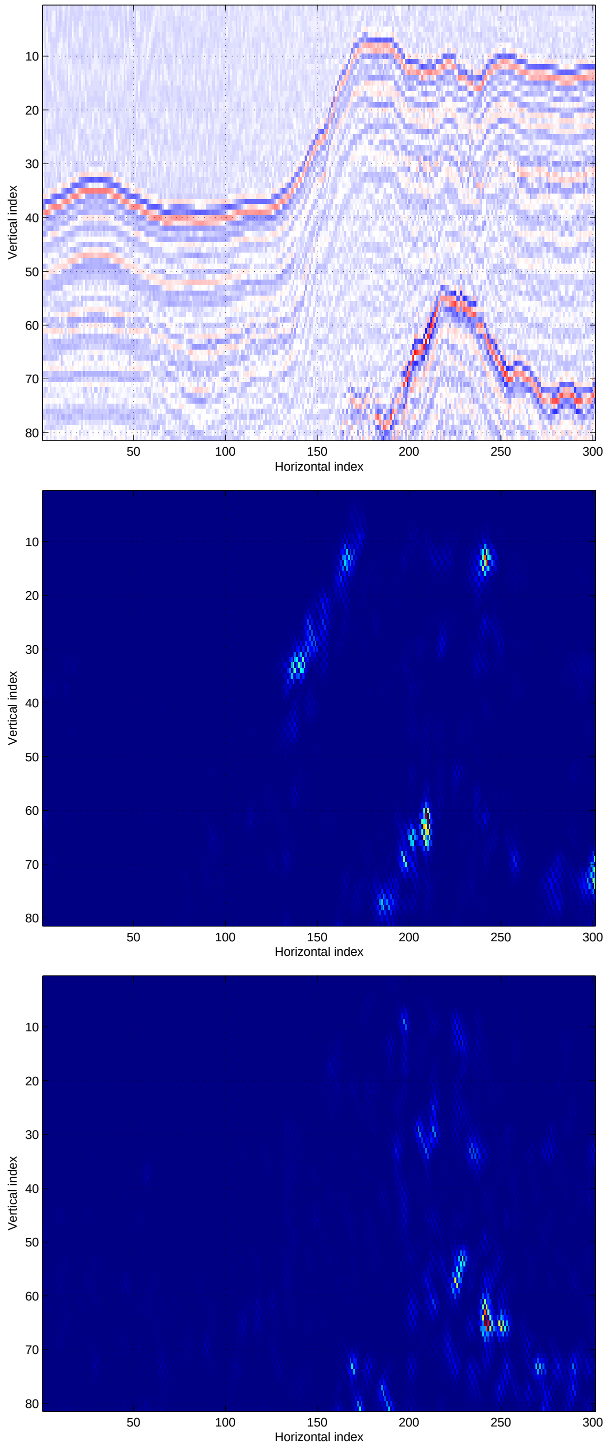
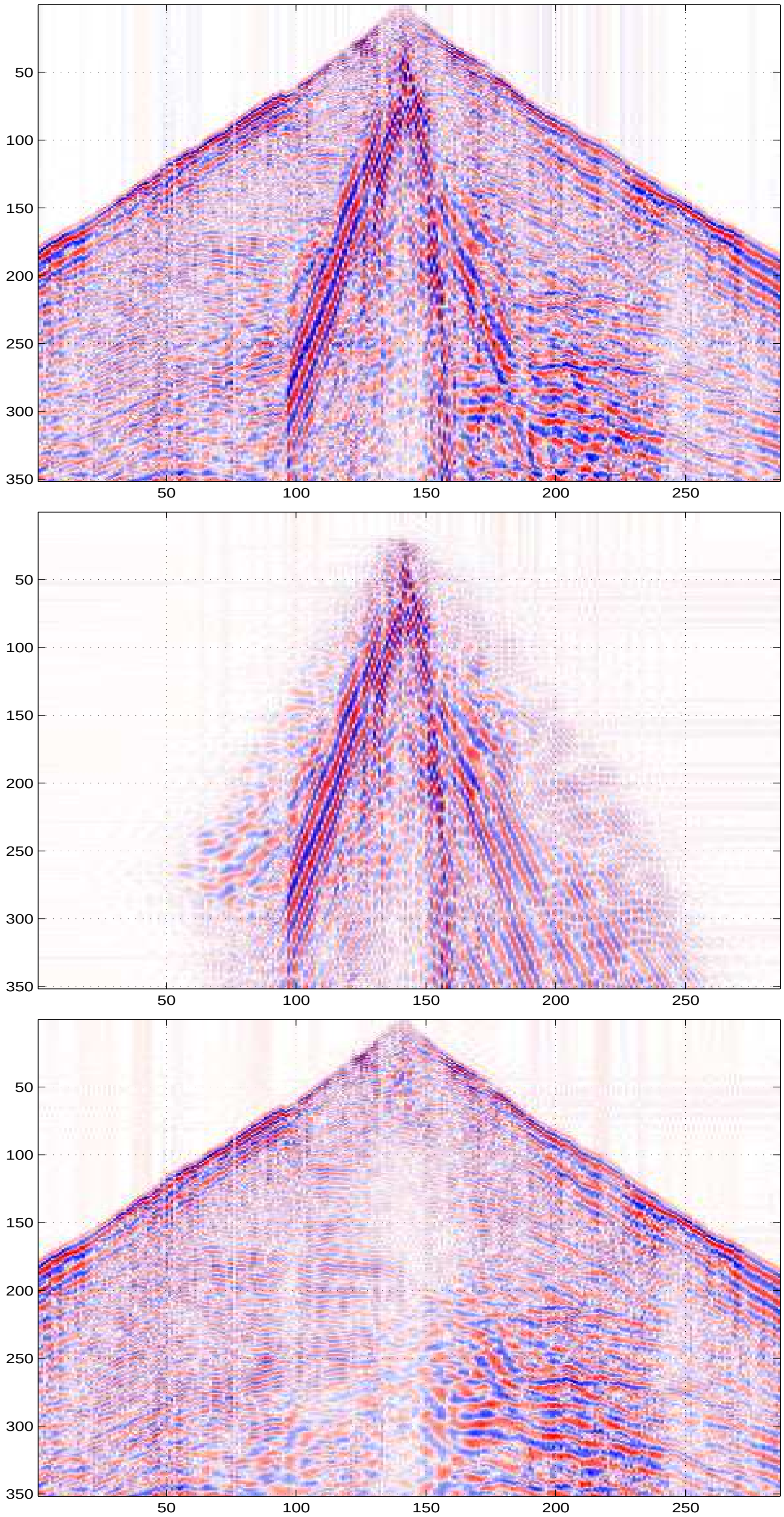


Figure 4: Local direction analysis: (Top) Seismic data; (Middle) Energy map after filtering in a positive direction with a primal dual-tree wavelet; (Bottom) Energy map after filtering in the opposite negative direction with a dual dual-tree wavelet .



(Bottom)

Figure 5: (Top) Original seismic data in shot gather; (Middle) Directional noise removed from local mute of dual-tree coefficients; (Bottom) Filtered residual.