Z99 EFFICIENT COHERENT NOISE FILTERING: AN APPLICATION OF SHIFT-INVARIANT WAVELET DENOISING

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Abstract

Coherent noise or surface waves filtering represents one of the most complex issues in land seismic data processing. Wavelet based filtering has recently begun to challenge the popular and robust frequency-wavenumber (f-k_x-k_y) filter. Wavelet filters provide fine time-scale representations and non linear filtering capabilities that yield in some instances better results on dispersive coherence noise. We propose in this work an improvement over the classical discrete wavelets filtering via the use of *shift-invariant wavelets*. Though relatively computationally expensive, their theoretical framework enables a closer approximation to the continuous wavelets, which results in finer filtering, less subject to aliasing and to wavelet ringing artifacts. Results are demonstrated on real seismic data sets. Improvements on ground-roll filtering show shift-invariant wavelets to be promising denoising techniques.

Introduction

Wavelet based techniques have emerged as tools capable of performing efficient separation and filtering of noises arising from different kinds of sources, ubiquitous in field seismic data, see e.g. [Deighan et al., 1998] or [Miao and Cheadle, 1998]. When looking at the wavelet transform from the filter bank perspective [Duval and Røsten, 2000], some drawbacks of the discrete wavelet transform become clear when dealing with aliased seismic data, since wavelet domain filtering implicitely adds aliasing to the theoretically perfectly reconstructed data. In the first part, we briefly recall the principles of the discrete wavelet transform and introduce the idea and some theoretical aspects of shift-invariant wavelet. The second part compares the results obtained by the discrete and the newly introduced shift-invariant wavelet transform for ground-roll removal.

Discrete wavelet transform: the classical and the shift-invariant

Discrete wavelets: the classical wavelet transform

The now well established discrete wavelet transform (DWT) is composed of a cascade of elementary blocks, as shown in Fig. 1a. The analysis block first filters the input trace $x(t)$ by a low-pass filter h_0 and a high-pass filter h_1. The DWT is classicaly non-expansive, i.e. the number of output samples is about the same as the input signal. Each filter output is thus decimated (or subsampled) by 2, i.e. every second sample is removed. Theoretically, structural aliasing could result from decimation, since the Nyquist-Shannon condition is not fulfilled anymore. The basic wavelet theory states that when the four analysis filters h_0, h_1, g_0 and g_1 are properly chosen, not only aliasing is completely removed, but the output $\hat{x}(t)$ will also be strictly equal to x, up to the integer time delay l (i.e. a multiple of the sampling interval).

For the sake of clarity, let us briefly recall some underlying equations. Two equations govern the exact recovery of the input trace, where the notation $F(z)$ stands for the z-transform of the filter f:

$$G_0(z)H_0(z) + G_1(z)H_1(z) \quad = \quad 2z^{-l}, \qquad (1)$$

$$G_0(z)H_0(-z) + G_1(z)H_1(-z) \quad = \quad 0. \tag{2}$$

Equation 2 precisely states how filters should be jointly designed to completely avoid aliasing, when the analysis and synthesis stages are performed one after the other, without intermediate processing (i.e. the *optional filtering* box between analysis and synthesis in Fig. 1-(a) is void). One usually chooses pairs of filters following the relationships $G_0(z) = H_1(-z)$ and $G_1(z) = -H_0(-z)$ [Strang and Nguyen, 1996]. But in classical processing such as compression or filtering, one usually performs several operations between the two stages, such as filtering, fine sample or time-scale mute... If one inserts the filters r and s between the analysis and the synthesis stages, the anti-aliasing Eq. 2 becomes:

$$X(-z)H_0(-z)H_1(-z)[R(z^2) - S(z^2)] = 0. \tag{3}$$

Equation 3 is not easy to comply to, even more when considering cascading wavelet blocks.

Classical wavelet filtering via thresholding in the wavelet domain usually does less harm than in the time or Fourier domain because of the smoothness of the wavelets. Wavelets therefore yield good results with actual uncoherent noise [Miao and Cheadle, 1998] or coherent noise removal [Deighan et al., 1998].

But space domain aliasing sometimes exists in seismic data (e.g. in shot gathers) because of the receiver lines spacing. As a consequence, it is not satisfactory to add even more aliasing to the data in the wavelet domain.

Discrete wavelets: the shift-invariant wavelet transform

Since we may expect two close traces to share the same wavelet coefficients (up to a delay), the cause for aliasing could be dropped, i.e. we could use the elementary analysis block from Fig. 1a without the downsampling procedure (see Fig. 1b). As a result, the block becomes shift-invariant (SI), i.e. $x(t)$ and $x(t-d)$ transforms yield the same output, up to the integer delay d. The property holds for the overall SIWT transform too, since each block in the cascade is SI (shift-invariant). The resulting transform is called the shift-invariant wavelet transform (SIWT).

SIWT is naturally more computationally expensive than DWT. The DWT has a low complexity of order $O(N)$ (N being the number of samples for a 1-D signal) but fast algorithms for SIWT exist that are $O(N\log(N))$.

But the main drawback of SIWT is its memory/storage burden. Since undersampling is dropped, each elementary analysis block from Fig. 1b now produces twice the number of samples for a 1-D signal. If we cascade J elementary SI-wavelet blocks on a d-D signal (for instance $d = 3$ for 3-D seismic), the SIWT outputs on average $J(2^d - 1) + 1$ samples for one single input sample. For instance, a 4-level SIWT decomposition of a 3-D gather involves a volume of data about 30 times the initial one. Let us recall that for DWT, the volume after transformation is the same as the initial volume. It is nevertheless possible to manage calculations dimension by dimension to avoid or reduce the data in memory at once, but it increases the overall algorithm complexity.

So why should we use SIWT instead of the classical DWT? The SIWT theory have been established independently by several authors, e.g. [Coifman and Donoho, 1995] or [Pesquet et al., 1996], in the scope of random noise removal. In a few words, besides the absence of filter aliasing, the advantages of the SIWT lie in its redundant nature that gives a denser approximation to the continuous wavelet than the classical DWT. Moreover, SIWT is less subject to Gibbs phenomenons or ringing artifacts than the DWT.

So far, SIWT as not been often applied to coherent noise removal. The next chapter provides a comparison of ground-roll removal using DWT and SIWT.

Results: Comparison between DWT and SIWT ground-roll removal

Traditional surface waves removal methods include the popular f-k filter and its 3-D avatar, the f-k_x-k_y filter, to take in account 3-D acquisition geometries. We focus here on DWT and SIWT 2-D filtering. Several authors have assessed ground-roll removal in 2-D with the classical DWT [Deighan and Watts, 1997, Abdul-Jauwad and Khène, 2000]. We refer to [Galibert et al., 2002] for a 3-D point of view with a

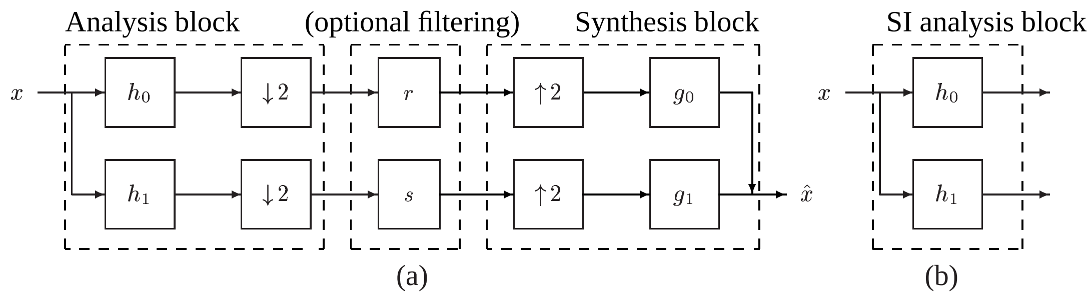

Figure 1: (a) Classical discrete wavelet filtering (b) Shift invariant elementary block.

comparison to the f-k_x-k_y filter.

Result 1: events continuity and linear waves filtering

Figure 2a shows a portion from a shot gather contamined with ground-roll. We use a Daubechies wavelet with the same time-scale selection for DWT and SIWT. Both DWT and SIWT filtering perform relatively well, and are able to retrieve weak reflection signals (Fig. 2b) burried under strong noise(Fig. 2c). We can see from the Fig. 3 close-up details that the SIWT performs slightly better than the DWT filter. Reflection waves show improved continuity and the linear waves are better eliminated.

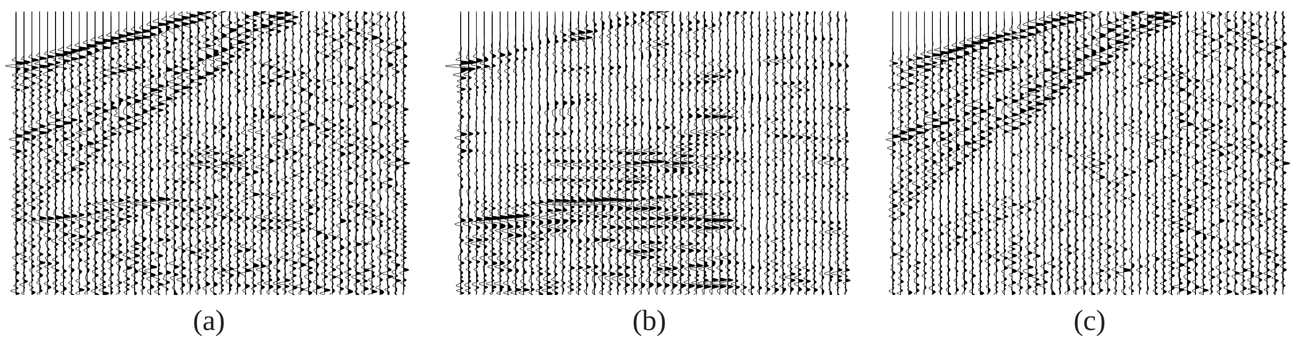

Figure 2: (a) Noisy data (b) SIWT filtered data and (c) cancelled noise with a Daubechies wavelet.

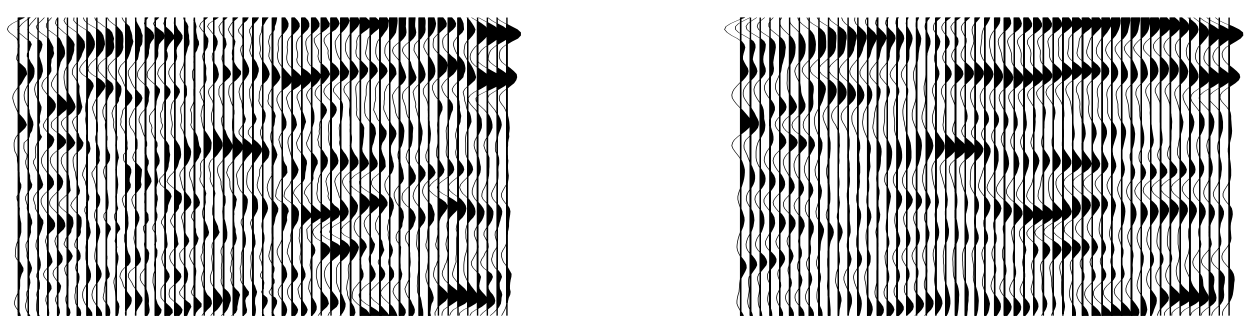

Figure 3: Zoom on DWT (left) and SIWT (right) filtering with a Daubechies wavelet.

Result 2: computation simplifications with the Haar wavelet

We already have pointed out that the SIWT is computionally expensive. Several methods have been devised to reduce its burden. It is well known that the longer a wavelet filter, the closer its frequency response is to an ideally sharp filter. The filter length adds to the computational complexity of the SIWT. The choice of a specific wavelet amongst the numerous wavelet families is not an obvious task. Classical applications

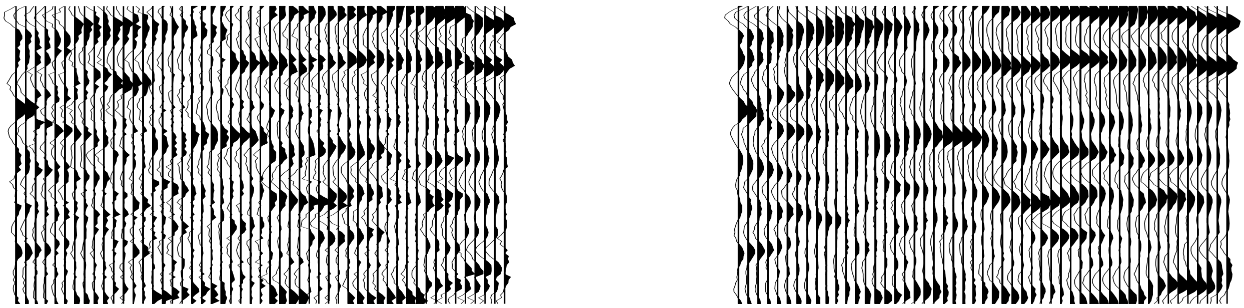

Figure 4: DWT (left) and SIWT (right) filtering with a Haar wavelet.

generally put the Haar wavelet aside, since it is often considered as trivial and inefficient for seismic signals (the Haar wavelet has only one zero-moment and its brick impulse response yields poor filters.) But it is very easy to compute since it requires only basic additions and substractions.

We show in Fig. 4 that the Haar wavelet performs indeed quite well in the SIWT framework, compared to the classical DWT. From Fig. 4 (right), we can see that the smoothness of the SIWT filtered signals is close to that of a more selective wavelet, as in Fig. 3b, while DWT results in Fig. 4 (left) are clearly unsatisfactory with the same poor wavelet.

Conclusions and perspectives

We have introduced the use of shift-invariant wavelet transforms (SIWT) for coherent noise filering. We have demonstrated on real 2-D data sets that the SIWT is able to outperform the DWT for ground-roll removal. We did not assess in the present work the management of 3-D irregularities originating from spatial sampling. These issues are discussed in [Galibert et al., 2002].

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